Journal of Applied Mechanics and Technical Physics, Vol. 36, No. 4, 1995

CONDITIONS IN A NODE OF ONE-DIMENSIONAL GAS FLOWS

Yu. A. Dubravin

UDC 532.542:536.73

In the general case of inner flows using a quasi-one-dimensional treatment of channel flows, hydrodynamic conservation laws in the integral form lead to an unclosed system of equations. Additional hypotheses are used to solve it, when dealing with problems on flow distribution from a channel (or inflow into a channel) through side branches. Thus, for example, Orlov and Mazing [1] assume a kinematic relation for the flows in the main channel and side branches, Petrov [2] uses hydrostatic relations for the inlet part of a side branch, Pavlov and Yaushev [3] replace the vector equation of motion by a scalar relation, having a hydraulic loss coefficient, and take the flow reducing ratio equal to unity in the side branch, when solving problems on decomposition of an arbitrary discontinuity on local resistances of different kinds; and Stepanov and Chudinov identify the supersonic stream efflux from the channel through the branch with the stream efflux from a half-infinite space through a slot in a thin plate [4].

A generalization of the hypothesis of [5] that the thermodynamic function (the coefficient of restoration σ_p) does not depend explicitly on the geometry of the channel and branches is used in the present work for solving the above-mentioned problem.

1. A channel segment with a generally variable transverse cross section within which an arbitrary number m of differently oriented branches are fit in a side wall is considered. The sections S^- and S^+ restrict the zone of flow rearrangement in the channel (Fig. 1), the sections S_{mk} of the branches are chosen where the assumption on one-dimensionality of the flow becomes acceptable (in the minimal section of the jet for the efflux case). Under the assumption used in [5], the hydrodynamic laws for the volume of the channel and branches between the mentioned sections yield the following equations:

$$(S\rho v)^+ = (S\rho v)^- - \sum_{k=1}^{m}(\varepsilon S\rho v_n)_{mk},$$

$$\begin{gathered}(\alpha S\rho v\mathbf{v} + S\mathbf{n}p)^+ - (\alpha S\rho v\mathbf{v} - S\mathbf{n}p)^- + p_\sigma \int\limits_{\sigma+\sum S_k} \mathbf{n}\,dS - \int\limits_{\sigma_t+S^-+S^+} \boldsymbol{\tau}_n^*\,dS \\ + \sum_{k=1}^{m}\Bigg\{(\alpha S\rho v_n^2)_{mk}\int\limits_{(\varepsilon S)_{mk}} \mathbf{n}\,dS + p_{mk}\int\limits_{(\varepsilon S)_{mk}} \mathbf{n}\,dS + p_{\delta k}\int\limits_{(1-\varepsilon)S_{mk}} \mathbf{n}\,dS + p_{\delta k}\Bigg[\int\limits_{S_{kA}} \mathbf{n}\,dS + \nu_k\int\limits_{\sigma_k} \mathbf{n}\,dS\Bigg] \\ + p_{\sigma k}\Bigg[\int\limits_{S_{kB}} \mathbf{n}\,dS + (1-\nu_k)\int\limits_{\sigma_k} \mathbf{n}\,dS\Bigg] - p_\sigma\int\limits_{S_k^-} \mathbf{n}\,dS - \int\limits_{\sigma_{tk}+S_{mk}} \boldsymbol{\tau}_n^*\,dS\Bigg\} = 0,\end{gathered} \qquad (1.1)$$

$$(S\rho vH)^+ - (S\rho vH)^- + \int\limits_{\sigma_t+S^-+S^+} (q_n^* - \boldsymbol{\tau}_n^*\cdot\mathbf{v})\,dS + \sum_{k=1}^{m}\Bigg\{(\varepsilon S\rho v_n H)_{mk} + \int\limits_{(\varepsilon S)_{mk}+\sigma_{tk}} (q_n^* - \boldsymbol{\tau}_n^*\cdot\mathbf{v})\,dS\Bigg\} = 0, \quad p = \rho RT.$$

Here p, ρ, v, T are the pressure, density, velocity, and temperature; $H = \beta v^2/2 + \gamma p/(\gamma-1)\rho$; α, β are the coefficients of heterogeneity of the velocity field; ε is the flow reducing ratio in side branches; $\mathbf{n}$ is the normal

Perm' State University, Perm' 614600. Translated from Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 36, No. 4, pp. 98–109, July–August, 1995. Original article submitted October 9, 1992; revision submitted August 2, 1994.

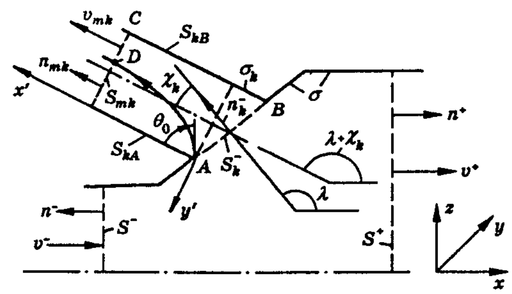

Fig. 1

vector; τ_n^*, $\mathbf{q}^*$ are the stress and heat flux vectors (including viscosity and turbulence effects); σ_t and σ_{tk} are the surfaces of the stream tubes for a transit flow and for that inflowing into or outflowing from the side branch. When writing down the equation of motion, a difference between the average pressure $p_{\delta k}$ in the flow separation zone and p_{mk} in the minimal section of the jet is permitted, which is confirmed by considerable pressure gradients in separation zones for the efflux of supersonic streams [6]; p_σ and $p_{\sigma k}$ are the average pressures on the side surface of the channel and the side surface of the kth branch (within the limits of the contact of the jet and branch surfaces) respectively. The regions of action of the branch surface pressures $p_{\delta k}$ and $p_{\sigma k}$ depend, in the general case, on branch orientation (the angle χ_k, see Fig. 1) and its shape. We assume for simplicity that the section S_{mk} is rectangular. Then the regions of action of these pressures can be described by means of the coefficient $\nu_k = (1 - \operatorname{sign}\chi_k)/2$, where the sign of χ_k is positive when the branch axis deviates counterclockwise from the normal to the surface σ. In this case, the lengths of the surface parts S_{kA} and S_{kB} are the same and can be determined as

$$b_k = (x'_D/a_m - \nu_1 \tan\chi)_k, \tag{1.2}$$

where $\nu_{1k} = \nu_k \operatorname{sign}\chi_k$; a_{mk} is the branch width, x'_{Dk} is the coordinate of the point D_k on the boundary of the jet minimal section (see below); $b_k = 0$ if the right-hand side of (1.2) is not positive.

The subsequent transformations of Eqs. (1.1) differ from [5] only by greater awkwardness and may be omitted. The only essential point is that the pressures p_σ and $p_{\sigma k}$ or their weight coefficients φ_p and φ_{pk} appear here as "superfluous" $m+1$ unknowns: $p_\sigma = \varphi_p p^+ + (1-\varphi_p)p^-$, $p_{\sigma k} = \varphi_{pk}p_{mk} + (1-\varphi_{pk})p_\sigma$. Then one can write the original equations as

$$\begin{gathered}(S\rho v)^+ = (1+\Delta_G)(S\rho v)^-, \qquad (1+\Delta_G)H^+ = H^- + \Delta_H(a^-)^2,\\ (\alpha S\rho v^2)^+ + p^+\sigma^+ = (\alpha S\rho v^2)^- + p^-\sigma^- - (S\rho v^2)\Delta_\tau^* - \sum_{k=1}^{m}(p^- - p_{\delta k})S_{mk}k_{3k};\end{gathered} \tag{1.3}$$

$$\varepsilon_{mk}(p_{mk} - p_{\delta k}) + (\varepsilon\rho v_n^2)_{mk}(\alpha_{mk} + \Delta_{\tau k}/n_{mk\tau})k_{1k} = k_{2k}(p_\sigma - p_{\delta k}), \qquad k = 1, m. \tag{1.4}$$

Equations (1.4) are the consequence of the equation of motion in a projection on the y- and z-axes; they permit the pressures p_{mk} to be eliminated from (1.3). Further on, Eqs. (1.4) are used for determining the unknowns Δ_{Gk}. The coefficients k_{ik} in (1.4) contain unknowns φ_{pk}, describe the form and orientation of branches, and, for the plane flows in branches, are reduced to the relations

$$\begin{gathered}k_{1k} = (1 + \varphi_{pk}\psi_{5k}/\varepsilon_{mk})^{-1}, \qquad k_{2k} = k_{1k}(\psi_{7k} - \psi_{5k} + \varphi_{pk}\psi_{5k}),\\ k_{3k} = k_{1k}(\psi_{1k} + \varphi_{pk}\psi_{2k}), \qquad k_{4k} = k_{1k}(\psi_{3k} + \varphi_{pk}\psi_{4k}/\varepsilon_{mk}),\\ \psi_{1k} = (-\tan\chi_k + q_k)/\sin(\lambda+\chi_k), \quad \psi_{2k} = (1-\varepsilon_{mk})q_k/\varepsilon_{mk}\sin(\lambda+\chi_k),\\ \psi_{3k} = \Delta_{\tau k}\cot(\lambda+\chi_k)/\alpha_{mk}, \ \psi_{4k} = [\alpha_k + \Delta_{\tau k}\sin(\lambda+\chi_k)]q_k/\alpha_{mk}\sin(\lambda+\chi_k),\\ \psi_{5k} = -\cot(\lambda+\chi_k)\cdot q_k, \ \psi_{6k} = q_k\sin(\lambda+\chi_k), \ \psi_{7k} = \sin\lambda/\cos\chi_k\sin(\lambda+\chi_k),\end{gathered} \tag{1.5}$$

$$q_k = b_k + (1-\nu_k)\tan\chi_k, \quad n_{mkr} = \sin(\lambda + \chi_k).$$

The remaining variables in (1.3), (1.4) are essentially similar to those in [5]:

$$\sigma^- = \sigma^+, \quad \sigma^+ = S^+[1-\varphi_p(\Delta_S - k_5)], \quad k_5 = (1-\Delta_S)\sum_{k=1}^m \overline{S}_{mk}k_{3k}, \quad \overline{S}_{mk} = S_{mk}/S^-, \quad \Delta_S = 1 - S^-/S^+,$$

$$\Delta_\tau^* = \Delta_\tau + \sum_{k=1}^m (\alpha_m \Delta_G v_{nm})_k k_{4k}/v^-, \quad \Delta_{Gk} = -G_k/G^-, \quad G_k = (\varepsilon S \rho v_n)_{mk}, \quad G^- = (S\rho v)^-, \quad \Delta_G = \sum_{k=1}^m \Delta_{Gk},$$

$$\Delta_Q = -\frac{\int\limits_{\sigma_t + S^- + S^+} (q_n^* - \boldsymbol{\tau}_n^* \cdot \mathbf{v})dS}{(Ga^2)^-}, \qquad \Delta_{Qk} = -\frac{\int\limits_{\sigma_{tk} + (\varepsilon S)_{mk}} (q_n^* - \boldsymbol{\tau}_n^* \cdot \mathbf{v})\, dS}{G_k(a^-)^2},$$

$$a^2 = \gamma p/\rho, \qquad \Delta_H = \Delta_Q + \sum_{k=1}^m \Delta_{Gk}(A_{Hk}(H/a^2)^- - \theta_k \Delta_{Qk}),$$

$A_{Hk} = 1$, $\theta_{Hk} = 0$ for ouflow from the channel, $A_{Hk} = H_k/H^-$, $\theta_{Hk} = 1$ for inflow into the channel,

$$\Delta_\tau = -\frac{\int\limits_{\sigma_t + S^- + S^+} \tau_{nx}^*\, dS + \sum\limits_{k=1}^m \int\limits_{\sigma_{tk} + S_{mk}} \tau_{nx}^*\, dS}{G^- v^-},$$

$\Delta_{\tau k}$, unlike Δ_τ, are connected with the transverse components of the stress vector for the stream tubes of side branches.

Equations (1.3) describe the reaction of the gas in the channel to local finite actions: geometric Δ_S, χ_k, flow rate Δ_{Gk}, thermal Δ_Q, Δ_{Qk} and friction forces Δ_τ, $\Delta_{\tau k}$. In the efflux problems under the given actions these equations are unclosed owing to $m+1$ unknown pressures p_σ, $p_{\sigma k}$ (or φ_p, φ_{pk}). In mass supply problems, where $p_{mk} = p_{\delta k} = p_{\sigma k} = p_\sigma$ [5], the group of integrals in braces in the equation of motion (1.1) vanishes except for the first and the last integrals by virtue of the known property $\int_{S_k} \mathbf{n}\, dS = 0$ (S_k is the closed volume surface of the kth side branch). In doing so, the number of unknowns φ_{pk} in Eqs. (1.3) reduces at the expense of branches with mass supply (to the only p_σ in the limit). In this case the values

$$k_{3k} = k_5 = 0, \quad \Delta_{\tau k} = -(\alpha n_\tau)_{mk}, \quad k_{4k} = -\cos(\lambda + \chi_k) \tag{1.6}$$

correspond to those in (1.3)–(1.5), which is in good agreement with the results [5] of mass supply to the channel; therefore the main consideration will be given to efflux problems.

2. Under the assumption that the gas state parameters in the secton S^- are known and in S^+ are sought for, solving hydrodynamic equations (1.3), carried out according to the scheme [5], results in the expressions

$$\begin{gathered}
\frac{v^+}{v^-} = \frac{K \pm \nu N}{(1+\Delta_G)(n+1)\mathrm{M}^{-2}}, \qquad \frac{\rho^+}{\rho^-} = \frac{(1-\Delta_S)(1+\Delta_G)^2(n+1)\mathrm{M}^{-2}}{K \pm \nu N}, \\
\frac{p^+}{p^-} = \frac{K \mp \nu n N}{I(n+1)}, \qquad \frac{T^+}{T^-} = \frac{(K \mp \nu n N)(K \pm \nu N)}{I(1-\Delta_S)(1+\Delta_G)^2(n+1)^2\mathrm{M}^{-2}}, \\
\mathrm{M}^{+2} = (1-\Delta_S) I \frac{K \pm \nu N}{K \mp \nu n N}, \quad \frac{p_\sigma}{p^-} = 1 + \varphi_p\Big(\frac{p^+}{p^-} - 1\Big), \quad \frac{p_{\sigma k}}{p_\sigma} = 1 + \varphi_{pk}\Big(\frac{p_{mk}}{p_\sigma} - 1\Big);
\end{gathered} \tag{2.1}$$

$$\begin{gathered}
\mathrm{M} = v/a, \quad \mathrm{M}^{-2} = (\mathrm{M}^-)^2, \quad n = (1 - (\gamma-1)\sigma^+/\gamma S^-)^{-1}, \quad I = n\sigma^+/\gamma S^-, \quad \nu = \mathrm{sign}\,(\mathrm{M}^- - 1), \\
K = I + n\mathrm{M}^{-2}(\alpha^- - \tilde{\Delta}_\tau^*), \quad N = \Big\{\frac{K^2 - 2(n^2-1)\mathrm{M}^{-2}(c_1 + c_2(\gamma-1)\mathrm{M}^{-2}/2)}{\gamma - 1}\Big\}^{1/2}, \\
c_1 = (1+\Delta_G)\Big(1 + \sum_{k=1}^m A_{Hk}\Delta_{Gk} + (\gamma-1)\Big(\Delta_Q - \sum_{k=1}^m \theta_{Hk}\Delta_{Gk}\Delta_{Qk}\Big)\Big),
\end{gathered} \tag{2.2}$$

$$c_2 = (1+\Delta_G)\Big(\beta^- + \sum_{k=1}^{m} A_{Hk}\Delta_{Gk}\Big), \quad \tilde{\Delta}_r^* = \Delta_r^* + \frac{\sum\limits_{k=1}^{m}(1 - p_{\delta k}/p^-)\overline{S}_{mk}k_{3k}}{\gamma M^{-2}}.$$

In relations (2.1), for the low sign choice, the possibility of normal shock is provided in addition to the actions mentioned above. Introducing the notation below for a part of the actions

$$X = \{x_i\} = \{M^-, \Delta_{Gk}, \Delta_Q, \Delta_{Qk}, \Delta_r, \Delta_{rk}\}, \tag{2.3}$$

the general structure of the solution for an arbitrary hydrodynamic parameter ψ can be presented in the form

$$\psi^+/\psi^- = f(X, \Delta_S, \chi_k, \varphi_p(X, \Delta_S, \chi_k), \varphi_{pk}(X, \Delta_S, \chi_k)).$$

Relations (2.1) are regarded to be an intermediate step of the solution because they have unknowns φ_p and φ_{pk} which are assumed to be functions of all possible actions.

3. The determination of φ_p and φ_{pk}, as in [5], is based on the possibility of describing the thermodynamic function $\sigma_p = p_0^+/p_0^-$ (p_0 is the drag pressure) in two ways. The first way (with the use of two thermodynamic laws written for a medium in the considered volume) yields the same relations as in [5] and suggests that the function $\sigma_p^{(1)}$ does not depend explicitly on the geometric characteristics of the channel (jump of the area of the channel section, orientation of side branches here) and is a function of the actions X

$$\sigma_p^{(1)} = f_1(X), \tag{3.1}$$

where the form of the function f_1 is not determined within the framework of the approach, and (3.1) has a qualitative character.

On the other hand, three conservation laws of hydrodynamics in the form of (2.1) give

$$\sigma_p^{(2)} = \frac{\pi(M^-)}{I(n+1)}(K \mp \nu n N)\Big(1 + \frac{n-1}{2}\frac{K \pm \nu N}{K \mp \nu n N}\Big)^{\gamma/(\gamma-1)} \tag{3.2}$$

$[\pi(M) = (\tau(M))^{\gamma/(\gamma-1)},\ \tau(M) = (1 + (\gamma-1)M^2/2)^{-1}]$, hence

$$\sigma_p^{(2)} = f_2(X, \Delta_S, \chi_k, \varphi_p(X, \Delta_S, \chi_k), \varphi_{pk}(X, \Delta_S, \chi_k)). \tag{3.3}$$

The dependence of the function f_2 on the arguments X, Δ_S, χ_k, and the variables φ_p and φ_{pk} is defined by relation (3.2).

Also, it is obvious that $\sigma_p^{(1)}$ and $\sigma_p^{(2)}$ determine one and the same quantity, hence

$$\sigma_p^{(1)} = \sigma_p^{(2)} \qquad \text{or} \qquad d\sigma_p^{(1)} - d\sigma_p^{(2)} = 0. \tag{3.4}$$

Then, the problem is reduced to the choice of the functions $\varphi_p(X, \Delta_S, \chi_k)$ and $\varphi_{pk}(X, \Delta_S, \chi_k)$, such that the function $\sigma_p^{(2)}$ is independent of Δ_S and χ_k. One of the possible ways of finding φ_p and φ_{pk}, which was used earlier in [5], is as follows: the differences of partial differentials of these functions and independence of the actions X, Δ_S, χ_k, substantiated in [5], yield a system of partial differential equations, which provides independence of the function $\sigma_p^{(2)}$ in (3.2) of geometic actions Δ_S, χ_k:

$$\frac{\partial f_2}{\partial \varphi_p}\frac{\partial \varphi_p}{\partial \Delta_S} + \sum_{k=1}^{m}\frac{\partial f_2}{\partial \varphi_{pk}}\frac{\partial \varphi_{pk}}{\partial \Delta_S} = -\frac{\partial f_2}{\partial \Delta_S}, \qquad \frac{\partial f_2}{\partial \varphi_p}\frac{\partial \varphi_p}{\partial \chi_j} + \sum_{k=1}^{m}\frac{\partial f_2}{\partial \varphi_{pk}}\frac{\partial \varphi_{pk}}{\partial \chi_j} = -\frac{\partial f_2}{\partial \chi_j}, \qquad j = 1, m. \tag{3.5}$$

Thus, relations (3.5) or the original condition (3.4) are analytical expressions of the hypothesis that the thermodynamic function σ_p does not depend explicitly on geometric actions taken as independent variables and extend previous results [5] to a greater number of actions. The solutions of (3.5) with respect to φ_p and φ_{pk}, together with (2.1), describe the reaction of the flow in the channel to local finite actions X, Δ_S, χ_k, forming a closed system of equations.

The investigation of the type of system of differential equations (3.5), carried out according to the scheme [7], has revealed that these equations are hyperbolic in the entire possible range of flow velocities and hence initial conditions must be specified to solve the system. In this case, any directions in the space of the variables Δ_S, χ_k are characteristic ones. Specifically, for m equal branches ($\chi_k = \chi$) the following relations hold:

$$\Delta_S = \text{const}: \quad m\frac{\partial f_2}{\partial \varphi_{pk}}\, d\varphi_{pk} + \frac{\partial f_2}{\partial \varphi_p}\, d\varphi_p = -m\frac{\partial f_2}{\partial \chi}\, d\chi,$$

$$\chi = \text{const}: \quad m\frac{\partial f_2}{\partial \varphi_{pk}}\, d\varphi_{pk} + \frac{\partial f_2}{\partial \varphi_p}\, d\varphi_p = -\frac{\partial f_2}{\partial \Delta_S}\, d\Delta_S.$$

4. Alternative methods of finding φ_p and φ_{pk} are examined below, first in the case of an incompressible liquid. Expanding the functions in Eqs. (2.1) and (3.2) in a series in powers of Mach numbers (in the vicinity of $\mathrm{M} = 0$ and retaining terms up to and including M^2) yields the relations

$$\frac{v^+}{v^-} = (1-\Delta_S)(1+\Delta_G), \quad \frac{p^+}{p^-} = 1+(1-\Delta_S)(\delta^*+c_1\Delta_S)/\mathrm{Eu}^-(1-\varphi_p(\Delta_S-k_5)),$$

$$\frac{p_\sigma}{p^-} = 1+\varphi_p(1-\Delta_S)(\delta^*+c_1\Delta_S)/\mathrm{Eu}^-(1-\varphi_p(\Delta_S-k_5)), \quad \delta^* = \alpha^- - \Delta_r^* - c_1,$$

$$\frac{p_{\sigma k}}{p^-} = \varphi_{pk}\frac{p_{mk}}{p^-} + (1-\varphi_{pk})\frac{p_\sigma}{p^-}, \quad \frac{\rho^+}{\rho^-} = 1, \quad c_1 = (1+\Delta_G)^2, \tag{4.1}$$

$$\mathrm{Eu}^- = 1/\gamma \mathrm{M}^{-2}, \quad \Delta_r^* = \Delta_r - \sum_{k=1}^{m} \alpha_{mk}\Delta_{Gk}^2 k_{4k}/\overline{S}_{mk}\varepsilon_{mk};$$

$$\sigma_p^{(2)} = \frac{2\mathrm{Eu}^- + 2(1-\Delta_S)(\delta^*+c_1\Delta_S)/(1-\varphi_p(\Delta_S-k_5)) + c_1(1-\Delta_S)^2}{1+2\mathrm{Eu}^-}, \tag{4.2}$$

which for $\chi_k = 0$ and $x'_{Dk} = 0$ ($k_{1k} = k_{2k} = 1$, $k_{3k} = k_{4k} = k_{5k} = 0$) become identical to those in [5]. For the latter we shall first indicate new ways of obtaining φ_p. In particular, as in [8], rearranging the variables that are dependent on the action Δ_S and independent of it (σ_p included) in the different parts of the equation allows the conclusion that both sides of the equation are a certain function $\lambda(X)$ of the actions X, the value of which is easily determined ($\lambda(X) = 0$ for $\Delta_S = 0$, hence $\lambda(X) = 0$ for any Δ_S):

$$\sigma_p(1+2\mathrm{Eu}^-) - 2\mathrm{Eu}^- - 2\delta - c_1 = \Delta_S[-2\delta - c_1\Delta_S + \varphi_p(\sigma_p(1+2\mathrm{Eu}^-) - 2\mathrm{Eu}^- - (1-\Delta_S)^2 c_1)].$$

This method gives values of σ_p and φ_p coinciding with the solution of the differential equation for φ_p [5], subject to the initial condition. In the second method, the hypothesis that $\sigma_p^{(2)}$ is independent of Δ_S is used and the values depending on Δ_S are excluded from the function f_2. As a result, the expression for σ_p ($\sigma_p = f_{20}$) is obtained immediately and the function φ_p is found in a finished form from the solution of the algebraic equation

$$f_2(X, \Delta_S, \varphi_p(X, \Delta_s)) - f_{20} = 0. \tag{4.3}$$

Both of the above methods give the same results. Equation (4.3) can be considered therewith as a partial integral of the differential equation for φ_p in [5].

The successive application of any of these methods to (4.2) gives

$$\sigma_p = \frac{2\mathrm{Eu}^- + 2\delta + c_1}{1+2\mathrm{Eu}^-}, \quad \varphi_p = \frac{c_1\Delta_S + 2\delta}{c_1\Delta_S(2-\Delta_S) + 2\delta}, \quad \delta = a^- - \Delta_r - c_1,$$

$$\varphi_{pk} = \varepsilon_{mk}[Q_{2k} - 2\Delta_{rk}\Delta_{Gk}^2\cos(\lambda+\chi_k)]/q_k[Q_{1k} + 2\Delta_{rk}\Delta_{Gk}^2\sin(\lambda+\chi_k)], \tag{4.4}$$

$$Q_{1k} = 2\alpha_{mk}\Delta_{Gk}^2 - (1-\varepsilon_{mk})\overline{S}_{mk}^2(c_1\Delta_S + 2\delta), \qquad Q_{2k} = \overline{S}_{mk}^2(c_1\Delta_S + 2\delta)(q_k - \tan\chi_k).$$

Relations (4.4) do not contain unknown functions, satisfy Eqs. (3.5) (this is verified by direct substitution), and hence can be considered as a partial integral of Eqs. (3.5).

Relations (4.4) leads to the essential (and natural) conclusion that the average pressures p_σ (or φ_p) at the channel side wall depend only on summary actions (Δ_G and Δ_r here) and are independent of local ones (in terms of belonging to an individual branch), χ_k, for example. Such a dependence can occur only implicitly due to the change in the total flow rate Δ_G (following the change in Δ_{Gk}) and Δ_τ. In turn, φ_{pk} depends on the characteristics of its "own" branch, and the rest of the branches affect it only through the total actions.

The obtained values φ_p and φ_{pk} make it possible to determine the functions in (1.5) for an incompressible liquid (for brevity, written out here for $\Delta_{\tau k} = 0$) in the form

$$k_{1k} = Q_{1k}\sin(\lambda + \chi_k)/Q_k, \quad k_{2k} = \{Q_{1k}\sin\lambda + \cos\chi_k\cos(\lambda + \chi_k)[q_k Q_{1k} - \varepsilon_{mk}Q_{2k}]\}/Q_k\cos\chi_k,$$
$$k_{3k} = 2\alpha_{mk}\Delta_{Gk}^2(q_k - \tan\chi_k)/Q_k, \quad k_{4k} = Q_{2k}/Q_k, \quad Q_k = Q_{1k}\sin(\lambda + \chi_k) - Q_{2k}\cos(\lambda + \chi_k)$$

and to find finally the hydrodynamic parameters (4.1) as functions of the actions.

5. The one-dimensional flow treatment for the efflux of an ideal liquid from a channel, beginning with the inlet section S_k^- of a side branch, results in flow "insensitivity" to the angle at which the mass is removed from the channel [5]. The abandonment of this assumption implies expansion of the boundaries of the fluid volume considered in the side branches up to the sections, where the assumption that the flow is one-dimensional becomes more justified, for example, in the minimal section of the jet in the separation zone (see Fig. 1). In this case, the channel flow parameters change with a change in the angle χ_k at which the mass is removed. The cost for the appearance of "sensitivity" of the flow is an increase from one to $m+1$ in the number of unknowns in the equations of hydrodynamics, and the action itself is realized by the values b_k (1.2) (or x'_{Dk}, see Fig. 1). An approximate method for finding the distance from the point of flow separation to the minimal jet section in a side branch for an ideal incompressible liquid is presented below.

The following assumptions are used: the flow in the branch is plane and nonturbulent in the vicinity of the boundary streamline AD, the flow in the minimal section is homogeneous and parallel to the axis of the branch, and the pressure $p_{\delta k}$ in the separation zone is constant and is the same as the pressure p_{mk} in the minimal jet section (therewith $v = v_{mk} = \text{const}$ on lines AD, DC). Under these conditions, the complex potential W and the complex velocity $\xi = (Q/va)_{mk}dW/dz$ can be introduced in the neighborhood of the boundary AD. Here W, ξ, $z = \bar{x}' + i\bar{y}'$ are normalized by the flow rate through the branch Q_{mk}, v_{mk}, and a_{mk} respectively. In the hodograph plane, an arc of a sector with an angle θ_0 corresponds to the streamline ψ_{AD}. Taking $\psi_{AD} = 0$ and using the definition of the complex velocity and the function [9]

$$W = (\xi + 1/\xi)/2,$$

which maps a circle arc in the plane ξ onto an interval on the real axis in the plane W, one can establish a relation $z(W)$ with the following real and imaginary parts:

$$\begin{gathered} 2\pi k - \theta_0 - \sin(2\pi k - \theta_0)\cos(2\pi k - \theta_0) = \beta 2(1 - \varepsilon_{mk})\varepsilon(\mathrm{M}_{mk})/\varepsilon_{mk}, \\ \bar{x}'_{Dk} = x'_{Dk}/a_{mk} = \varepsilon_{mk}\sin^2\theta_0/2\varepsilon(\mathrm{M}_{mk}), \quad k \in N. \end{gathered} \tag{5.1}$$

Here $\varepsilon(\mathrm{M}_{mk}) = 1$ for an incompressible liquid, $\beta = -1$ for $k \leqslant 0$ and $\beta = 1$ for $k > 0$ (a sequence of values k is chosen from the condition of monotone behavior of ε_{mk}); $\varepsilon_{mk} = (Q/va)_{mk}$ is the flow reducing ratio, related to the flow rate action as follows:

$$\Delta_{Gk} = -\varepsilon_{mk}\overline{S}_{mk}\mu_k, \quad \mu_k = v_{mk}/v^- = (1 + 2\mathrm{Eu}^-(1 - p_{mk}/p^-))^{1/2}. \tag{5.2}$$

Thus, the relations (5.1), (5.2) establish a link of x'_{Dk} with the flow rate action. In doing so, with the help of the circulation $2\pi k$, a "turbulent zone" in the vicinity of the entrance into the side branch is simulated. The occurrence of the zone is accompanied by a flow rate drop in the branch [10, 11].

On the whole, the relations in Secs. 4 and 5 make it possible to determine explicitly the state parameters of the channel flow of an incompressible liquid under any local actions (being taken into account here). In some cases, experimental data on hydraulic loss ζ_3 in fitted triple-branches [12] can be useful for taking into account the actions Δ_τ. Thus, for a flow in a channel, as in [5], we have

$$\zeta_3 = 1 - c_1 - 2\delta = 2(1 - \alpha^-) + \Delta_G(2 + \Delta_G) + 2\Delta_\tau. \tag{5.3}$$

6. In the case of a compressible liquid, an effective way of finding σ_p, φ_p, and φ_{pk} is the last approach considered by the method of Sec. 4, which leads to the following relations (the case of identical branches is considered for brevity):

$$\sigma_p = f_{20}(X) = \frac{\pi(\mathrm{M}^-)}{\gamma+1}(K_0 \mp \nu\gamma N)\Big(1+\frac{\gamma-1}{2}\,\frac{K_0 \pm \nu N_0}{K_0 \mp \nu\gamma N_0}\Big)^{\gamma/(\gamma-1)},$$
$$K_0 = 1+\gamma \mathrm{M}^{-2}(\alpha^- - \Delta_\tau), \quad N_0 = \{K_0^2 - 2(\gamma+1)\mathrm{M}^{-2}(c_1 + c_2(\gamma-1)\mathrm{M}^{-2}/2)\}^{1/2}; \tag{6.1}$$

$$f_2^0(X,\Delta_S,\varphi_p(X,\Delta_S)) - f_{20}(X) = 0, \quad f_2^0 = \frac{\pi(\mathrm{M}^-)}{I^0(n^0+1)}(K^0 \mp \nu n^0 N^0)\Big(1+\frac{n^0-1}{2}\,\frac{K^0 \pm \nu N^0}{K^0 \mp \nu n^0 N^0}\Big)^{\gamma/(\gamma-1)},$$
$$K^0 = I^0 + n^0\mathrm{M}^{-2}(\alpha^- - \Delta_\tau), \quad N^0 = \{K^{0^2} - 2(n^{0^2}-1)\mathrm{M}^{-2}(c_1 + c_2(\gamma-1)/2)/(\gamma-1)\}^{1/2}, \tag{6.2}$$
$$n^0 = \gamma(1+(\gamma-1)\varphi_p\Delta_S)^{-1}, \quad I^0 = (1-\varphi_p\Delta_S)/(1-\Delta_S)(1+(\gamma-1)\varphi_p\Delta_S);$$

$$f_2(X,\Delta_S,\chi_k,\varphi_p(X,\Delta_S),\varphi_{pk}(X,\Delta_S,\chi_k)) - f_{20}(X) = 0. \tag{6.3}$$

The transcendental Eqs. (6.2) and (6.3) are solved sequentially to find $\varphi_p(X,\Delta_S)$ and $\varphi_{pk}(X,\Delta_S,\chi_k)$. The independence of φ_p from the local actions χ_k (see Sec. 4) is taken into account in these equations.

When determining x'_{Dk} for a compressible flow, in addition to and partly instead of the assumptions made earlier (Sec. 5), the following assumptions are used: the flow is stationary, the processes are adiabatic in the neighborhood of the streamline AD, $\mathrm{M}^- < 1$, $\mathrm{M}_{mk} \leqslant 1$ (supersonic flows in the neighborhood of the branch entrance have curvilinear shocks [6] and require a special consideration). Under these conditions, the velocity potential and the flow function are related as follows [13]:

$$\frac{\partial\varphi}{\partial x} = \frac{\rho_0}{\rho}\,\frac{\partial\psi}{\partial y}, \qquad \frac{\partial\varphi}{\partial y} = -\frac{\rho_0}{\rho}\,\frac{\partial\psi}{\partial x}.$$

Here $\rho/\rho_0 = \rho_{mk}/\rho_0 = \varepsilon(\mathrm{M}_{mk}) = \mathrm{const}$ [$\varepsilon(\mathrm{M})$ is a gasdynamic function for the density] and in the variables (x,y) can be identified with the Chaplygin function. Introducing the reduced velocity potential $\varphi_1 = \varphi\varepsilon(\mathrm{M}_{mk})$, one can reduce the problem being examined to the earlier-solved problem (5.1), (5.2), where $\varepsilon(\mathrm{M}_{mk}) < 1$ and μ_k is defined in [5]:

$$\mu_k = (\mathrm{M}_{mk}/\mathrm{M}^-)(\tau(\mathrm{M}_{mk})/\tau(\mathrm{M}^-))^{(\gamma+1)/2(\gamma-1)}.$$

7. The relations obtained, as applied to the problems of outflow from a channel (inflow into a channel), make it possible to state and solve a range of problems: determination of flow rates (taking into account jet reduction) for an efflux of incompressible and compressible liquids from a channel through an arbitrary oriented branch into a medium with a given pressure, into a closed volume, an injection of a liquid into a channel, an overflow of a liquid between crossed channels through a connective branch, finding the boundaries of these regimes of flow, theoretical values of coefficients of hydraulic losses [see (5.3) for $\Delta_\tau = 0$], etc.

In particular, an efflux of a compressible liquid from a channel through a branch ($m = 1$) into a medium with a given pressure $p_{\delta k}$ is described by Eqs. (1.4) for $p_{mk} = p_{\delta k}$, by (1.5), by a part of relations (2.1) (choosing the upper sign) for the pressures p_σ and p^+, by (5.1), and (6.1)–(6.3). The listed equations allow revealing the influence of the branch orientation χ_k on the channel flow rate Δ_G under the other given actions Δ_S, Δ_τ, $\Delta_{\tau k}$, Δ_Q.

In the case of an ideal ($\Delta_\tau = \Delta_{\tau k} = 0$, $\alpha^- = 1$, $\alpha_{mk} = 1$), incompressible [$\varepsilon(\mathrm{M}_{mk}) = 1$] liquid, effluxing through a side branch of a channel with a constant section [$\Delta_S = 0$, $\varphi_p = 1$, see (4.4), $p_\sigma = p^+$, see Sec. 1], the original system of equations is significantly simplified and consists of: (1.4) for $p_{mk} = p_{\delta k}$; (4.1) for p_σ, where $\delta^* = 1 - c_1 - \Delta_\tau^*$ and $\Delta_\tau^* = -\Delta_G^2 k_{4k}/\overline{S}_{mk}\varepsilon_{mk}$, $c_1 = (1+\Delta_G)^2$; (4.5) for k_{1k}, k_{2k}, k_{4k}; (4.4) for Q_{1k}, Q_{2k}, where $q_k = b_k + (1-\vartheta_k)\tan\chi_k$ and $q_k = b_k + (1-\vartheta_k)\tan\chi_k$, $b_k = \bar{x}'_{Dk} - \vartheta_{1k}\tan\chi_k$ and ϑ_k and ϑ_{1k} see in Sec. 1; (5.2) connects the flow rate Δ_G and flow reducing ratio ε_{mk}; (5.1) connects $\bar{x}'_{Dk}$ with the flow rate Δ_G through an intermediate parameter θ_0 [θ_0 is the deviation angle of the boundary streamline AD at point A of the flow separation (see Fig. 1)]. It is convenient to reduce all the above equations to a system of two equations with respect to θ_0 and ε_{mk}. One of them [Eq. (5.1)] remains unmodified and allows the function

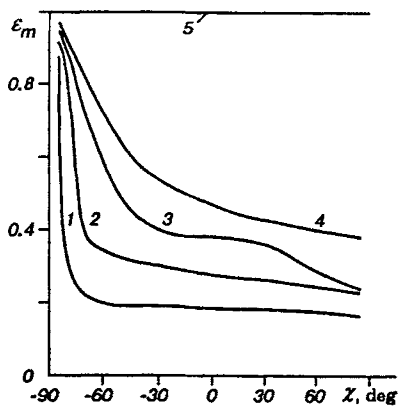

Fig. 2

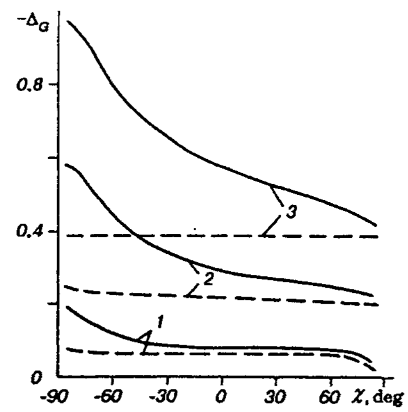

Fig. 3

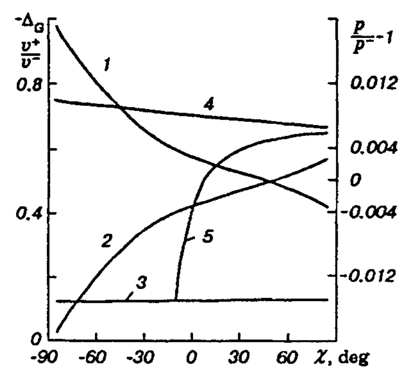

Fig. 4

$\theta_0(\varepsilon_{mk})$ to be found, and the remaining one can be reduced to an equation with respect to ε_{mk} (the index k is omitted below):

$$\varepsilon_m^2 \overline{S}_m^2 \mu^2 - (2\overline{S}_m\mu - \mu^2)\varepsilon_m - F(\varepsilon_m, \theta_0, \chi, \overline{S}_m, \mu) = 0. \qquad (7.1)$$

Here

$$F = (\mu^2 - 1)/2 + f(2\overline{S}_m\mu\varepsilon_m - \overline{S}_m^2\mu^2\varepsilon_m^2 + (\mu^2 - 1)/2);$$

$$f = \tan\chi \, \frac{(1 + \overline{S}_m^2)\mu\varepsilon_m^2 - 2\overline{S}_m\varepsilon_m}{2\overline{S}_m + \mu\overline{S}_m^2\varepsilon_m^2 - [2\overline{S}_m + (1 + \overline{S}_m^2)\mu]\varepsilon_m} \, \frac{\sin^2\theta_0}{2}.$$

The result of solving the above equations is the flow reducing ratio ε_{mk} represented as a function of the geometric characteristics of a side branch χ_k, $\overline{S}_{mk} = S_{mk}/S^-$ and of the complex hydrodynamic parameter $\mu_k = v_{mk}/v^- = (1 + 2\mathrm{Eu}^-(1 - p_{mk}/p^-))^{1/2}$, $\mathrm{Eu}^- = p^-/\rho(v^-)^2$. The found values ε_{mk} make it possible to calculate the hydrodynamic parameters in the channel output section and the average jet pressure on the wall (CB, see Fig. 1) of the side branch (4.1), provided that Δ_G, k_{3k}, k_5, k_{4k} from (4.5), and Δ_τ^*, φ_{pk} from (4.4) are predetermined.

One can state a problem on the efflux of an incompressible liquid from a channel through a long side branch, in which the flow separation zone is isolated from the environment and the hydraulic losses ζ_1 are significant, if the Bernoulli integral for the real liquid and the equation of continuity are added to the above-mentioned equations.

Some of the calculation results are given in Figs. 2–4. A growth in the velocity ratio $\mu_k = v_{mk}/v^-$

increases the "sensitivity" of the flow to the angle at which mass is removed through a short branch (function $\varepsilon_m(\chi)$ in Fig. 2, where $\overline{S}_m = 0.1$ and $\mu_k = 1.2$, 1.4, 1.8, 3 for curves *1–4* respectively; ε_m is given for comparison by curve *5* according to the hypothesis of [3]). For an incompressible fluid efflux through a side branch from a semi-infinite space the variable μ_k gives the same effect [11]. A greater "sensitivity" to the angle at which mass is removed reveals itself with a growth in the size of the opening $\overline{S}_m$ (function $\Delta_G(\chi)$ in Fig. 3, where $\mu_k = 2$ and $\overline{S}_m = 0.1$, 0.3, 0.5 for curves *1–3* respectively, the solid curves correspond to an efflux from a short branch, and the dashed curves from a long one with $\zeta_l = 5$). Therewith, an increase in the hydraulic losses in a side branch (with an increase in its length) is accompanied by a decrease in the flow rate and "sensitivity" to the angle at which mass is removed from the channel. For the cases examined, the "sensitivity" almost disappears for $\zeta_l = 5$ (which corresponds to the branch length in calibers $\sim 10^2$ for the quadratic resistance law). These results agree with the experiment of [2], where for an efflux through a long branch under the conditions $\overline{S}_m = 0.0036$–0.142 and $-50° \leqslant \chi_k \leqslant 40°$ the variation $\Delta_G(\chi)$ of the flow rate did not fall outside the limits of the "error, permissible for practical purposes." Detailed characteristics of one typical variant [$\overline{S}_m = 0.5$, $\mu_k = 2$, $p_{mk}/p^- = 0.985$, and the branch is short: (*1*) Δ_G, (*2*) v^+/v^-, (*3*) $p_{mk}/p^- - 1$, (*4*) $p_\sigma/p^- - 1$, (*5*) $p_{\sigma k}/p^- - 1$] are given in Fig. 4. In this case, the flow rate drops down when the branch axis deviates toward the flow, the pressure $p_\sigma = p^+$ decreases, and $p_{\sigma k} \to p_\sigma$ when $\chi_k \to \pi/2$. In certain cases, Eq. (7.1) has an explicit form of the solution, e.g., for $\chi_k = 0$, it is reduced to a quadratic equation (as in [5] for a compressible liquid); if we assume additionally $\overline{S}_m \to 0$ (an efflux from an infinite half-space), then $\varepsilon_m = (\mu^2 - 1)/2\mu^2$, whence $\varepsilon_m = 0$ for $\mu = 1$ is the same result as in [10], and $\varepsilon_m \to 1/2$ for $\mu \to \infty$ [14] (an efflux through a nozzle from a semi-infinite volume with a stagnant liquid), as might be expected.

A generalization of the hypothesis [5] to a greater number of geometric actions, carried out in the present work, allows the conclusion that the store of mechanical energy of a flow, characterized by the pressure restoration coefficient σ_p, does not change under any geometric actions, considered as independent arguments of the function σ_p. Thus, one can state and solve, in addition to the above-mentioned problems, the problems on finding the gas state parameters in channels with axis breaks and with other possible combinations of geometric actions. The problems of mass supply, where $p_\sigma = p_{\delta k} = p_{mk}$, are somewhat specific. Then it follows from (1.4) that $\Delta_{\tau k} = -(\alpha n_\tau)_{mk} = \alpha_{mk} \sin(\lambda + \chi_k)$, i.e., the whole momentum, which is normal to the flow axis, is taken by friction forces [5]. The variable $\Delta_{\tau k}$ is in the list of arguments (2.3) of the function σ_p (3.1). Hence the angles of mass supply are excluded from the list of geometric actions due to the unique relation with $\Delta_{\tau k}$.

On the whole, relations (2.1), (6.2), and (6.3) or their analogues for an incompressible liquid can be considered as conditions in a node of one-dimensional liquid flows that satisfy all hydrodynamic conservative laws, the second law of thermodynamics included.

REFERENCES

1. B. V. Orlov and G. Yu. Mazing, *Thermodynamic and Ballistic Fundamentals of the Design of Solid Fuel Rocket Engines* [in Russian], Mashinostroenie, Moscow (1979).
2. G. A. Petrov, *Motion of Liquid with Rate Change along the Way* [in Russian], Stroiizdat, Moscow–Leningrad (1951).
3. S. V. Pavlov and I. K. Yaushev, "Problem on the decomposition of an arbitrary discontinuity of gas parameters in branching channels," in: Numerical Analysis [in Russian], Novosibirsk (1978).
4. I. R. Stepanov and V. I. Chudinov, *Some Problems of Gas and Liquid Flows in Channels and Tubes of Power Settings* [in Russian], Nauka, Leningrad (1977).
5. Yu. A. Dubravin, "Relation between the hydrodynamic parameters in zones of local disturbances of the flow," *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 3, 60–69 (1989).
6. L. S. Plevako, L. V. Marchenko, et al., *Intermediate Ballistics of Artillery Guns* [in Russian], TsNII Informatsii, Moscow (1982).
7. S. K. Godunov, *Equations of Mathematical Physics* [in Russian], Nauka, Moscow (1971).

8. S. V. Vallander, *Lectures on Hydroaeromechanics* [in Russian], Leningrad Gos. Univ., Leningrad (1978).
9. G. K. Korn and T. K. Korn, *Handbook of Mathematics* [in Russian], Nauka, Moscow (1973).
10. E. N. Bogomolov, "Side efflux of liquid from a flow through an opening of finite depth," *Prikl. Mekh.*, **4**, No. 10 (1968).
11. E. N. Bogomolov, "Nonturbulent efflux of liquid from a flow through a side opening of finite depth," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 1 (1977).
12. I. E. Idel'chik, *Handbook of Hydraulic Resistances* [in Russian], Gosenergoizdat, Moscow–Leningrad (1960).
13. L. I. Sedov, *Plane Problems of Hydrodynamics and Aerodynamics* [in Russian], Nauka, Moscow (1980).
14. I. P. Ginzburg, *Applied Hydro- and Gas-Dynamics* [in Russian], Leningrad. Gos. Univ., Leningrad (1958).